Quasiscarred Resonances in a Spiral-shaped Microcavity and Fresnel Filtering Effects

Soo-Young Lee, Muhan Choi, Chil-Min Kim

NCRI Center for Controlling Optical Chaos,
Pai-Chai University, Daejeon 302-735, Korea
pmzsyl@yahoo.co.kr

Abstract

We discuss the quasiscarring phenomenon in a spiral-shaped dielectric microcavity. Unlike the scarred resonances which occur in many different chaotic systems, in the quasiscarred resonance there is no unstable periodic orbit underlying the pattern of localization. We analyze the Fresnel filtering effects on the quasiscarred resonances and show that the theoretical expectation explains qualitatively the quasiscarred resonance patterns.

Scars, the quantal imprints of unstable periodic orbits embedded in the corresponding classical dynamics, is a very intriguing and popular topic in the study of quantum chaos[1]. This scarring phenomenon has been studied extensively mainly in billiard system due to its simplicity[2]. Later, it has been observed in various chaotic systems such as microwave cavities, semiconductor quantum wells, surface waves, optical cavities, etc.[3, 4, 5, 6, 7, 8] We note that the chaotic systems are not closed, but open, different from billiard systems. This substantial difference might give a possibility that the scarring phenomenon can be changed or generalized in open systems according to the characteristics of the openness.

In our previous paper[9], we pointed out that in a spiral-shaped microcavity resonance patterns have strong localization of simple geometries, triangle and star shapes for $n = 2$ and 3 cases, respectively, where n is the reflective index of the microcavity. Unlike the scarred resonance there is no unstable periodic orbit underlying the localization pattern. This surprising finding implies that the localization effect originates from more complicated interplay among lights, boundary geometry, and material properties than the common scar theory for closed systems.

In this paper, we discuss on the Fresnel filtering effects on the reflected beam from a planar dielectric interface, and give a qualitative explanation for the quasiscarred resonance pattern in a spiral-shaped dielectric microcavity. This qualitative agreement means that the properties of openness of systems are crucial to understand quasiscarred resonance patterns.

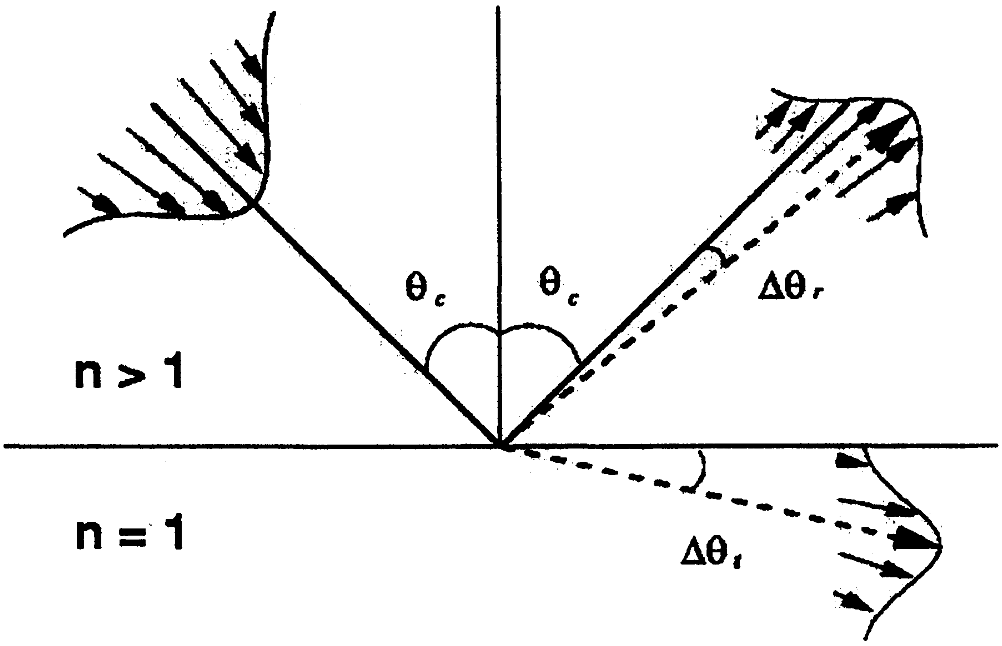

Figure 1: The conceptive explanation for the Fresnel filtering effects at a planar dielectric interface.

The conceptive explanation for the Fresnel filtering effects on reflected and transmitted beams is described in Fig.1. Consider a incident beam with a Gaussian profile of incident angles of plane waves, and assume that the central incident angle of the beam is the critical angle for total internal reflection, i.e., $\theta_c = \arcsin(1/n)$. Then, a half of the plan wave components has incident angles less than the critical angle and they are partially reflected and the rest is transmitted based on the Fresnel equation. On the other hand, the incident angles of the other half of plane waves are greater than the critical angle and they are totally reflected as shown in Fig.1. This process on the interface results in the angular shift of the transmitted beam, $\Delta\theta_t$ and the angular shift of the reflected beam, $\Delta\theta_r$. As mentioned later, the Goos-Hänchen lateral shift also appears for the reflected beam and this originates from the variation of the phase shift of totally reflected plane wave components.

Tureci and Stone[10] have studied the Fresnel filtering effect on the transmitted beam using a saddle point approximation. They considered the incident beam as a Gaussian angular profile of plane waves,

$$G(s) \propto \exp\left[-(\frac{\Delta}{2})^2 s^2\right], \tag{1}$$

where $s = \sin(\delta\theta_i)$ and $\delta\theta_i$ is the deviation angle of the plane wave component from θ_c. Their result for the angular shift of the transmitted beam is

$$\Delta\theta_t \simeq (2/\tan\theta_c)^{1/2}\Delta^{-1/2}. \tag{2}$$

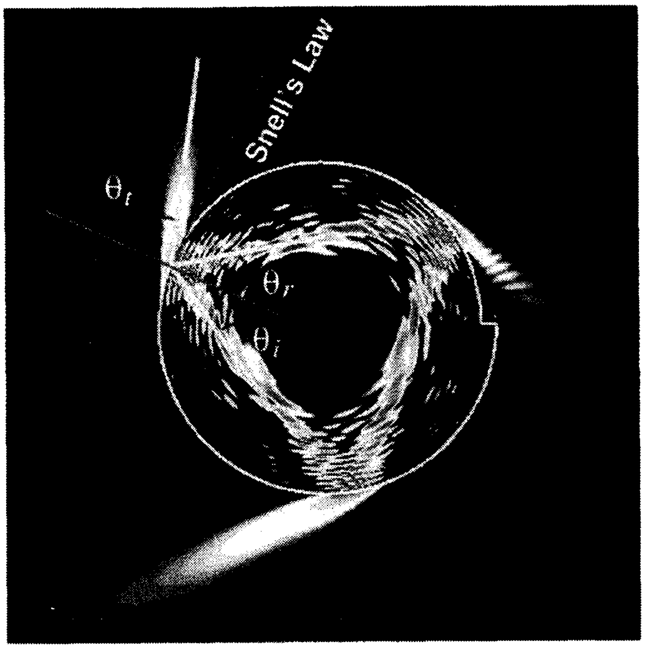

Figure 2: A quasiscarred resonance pattern in a spiral-shaped microcavity.

This result explains qualitatively the lasing emission from a scarred modes in a deformed microcavity[8].

Lai et al.[11] have investigated the Goos-Hänchen lateral shift around the critical angle and obtained the finite lateral shift at the critical angle. However they did not discuss about the angular shift of the reflected beam. Now, we extend their theory and estimate the angular shift $\Delta\theta_r$ of the reflected beam. The Field of the reflected beam can be expressed as

$$E_r(\rho, \phi) \propto \int ds\, R(s)\, G'(s)\, \exp\left[ink\rho\cos(\phi - \theta_i - \delta\theta_i)\right], \tag{3}$$

where ρ is the distance from origin and ϕ being the angle from the perpendicular axis from the planar interface. The reflection amplitude $R(s)$ is given by the Fresnel equation. In the approximation of $s \ll 1$ and when the peak of the Gaussian angular profile is at the critical angle, the reflection amplitude can be approximated as

$$R(s) \simeq 1 - \frac{1}{\cos^2\theta_c}\left[2\cos\theta_c\sqrt{\sin 2\theta_c}\sqrt{-s} + 2\sin 2\theta_c s + 2\sin\theta_c\sqrt{\sin 2\theta_c}s\sqrt{-s}\right]. \tag{4}$$

The Gaussian profile has additional phase factor, i.e.

$$G'(s) = G(s)\exp(ink\sqrt{1-s^2}\,z_0), \tag{5}$$

where z_0 is the distance between origin and the position where the incident beam has a minimum beam waist. Toghter with the relation

$$\cos(\phi - \theta_i - \delta\theta_i) \simeq \cos(\phi - \theta_i)(1 - \frac{1}{2}s^2) + \sin(\phi - \theta_i)s, \tag{6}$$

we can integrate Eq.(3) and the result can be expressed in terms of the parabolic-cylinder functions. The resulting field intensity of the reflected beam with $z_0 = 10$, $\theta_c = \pi/6$, and $\Delta = 17$ explains well the Goos-Hänchen shift and the angular shift $\Delta\theta_r$.

In Fig. 2 a quasiscarred resonance with $n = 2$ is shown. The resonance shows high chirality, i.e., only clock-wise rotating waves are dominant. In this situation, we can estimate various angles such as incident angle θ_i, reflection angle θ_r, and transmission angle θ_t. These are $\theta_i \simeq 30^o$, $\theta_r \simeq 32.1^o$, and $\theta_t \simeq 74.5^o$. This means that $(\Delta\theta_r)_{QS} \simeq 2.1^o$ and $(\Delta\theta_t)_{QS} \simeq 15.5^o$. These values can be compared with the theoretical results, $(\Delta\theta_r)_{TH} \simeq 0.8^o$, $(\Delta\theta_t)_{TH} \simeq 25.8^o$, and $\Delta l_{GH} \simeq 0.5\lambda$.

In conclusion, although the theory for the Fresnel filtering effects, based on assumptions of Gaussian angular component of plane waves and planar interface, is not enough to explain the quasiscarred resonance patterns in a Spiral-shaped microcavity, there is qualitative agreement between them. For the quantitative agreement, we need to know the effect of curved interface and realistic beam profile.

This work is supported by Creative Research Initiatives of the Korean Ministry of Science and Technology.

References

[1] E. J. Heller, Phys. Rev. Lett. **53**, 1515 (1984).

[2] S. C. Creagh, S.-Y. Lee, and N. D. Whelan, Ann. Phys. **295**, 194 (2002); S.-Y. Lee and S. C. Creagh, Ann. Phys. **307**, 392 (2003).

[3] S. Sridhar, Phys. Rev. Lett. **67**, 785 (1991); S. Sridhar and E. J. Heller, Phys. Rev. A **46**, R1728 (1992).

[4] T. M. Fromhold et al., Phys. Rev. Lett. **75**, 1142 (1995).

[5] A. Kudrolli et al., Phys. Rev. E **63**, 026208 (2001).

[6] S.-B. Lee et al., Phys. Rev. Lett. **88**, 033903 (2002).

[7] T. Harayama et al., Phys. Rev. E **67**, 015207(R) (2003).

[8] N. B. Rex et al., Phys. Rev. Lett. **88**, 094102 (2002).

[9] S.-Y. Lee et al., Phys. Rev. Lett. **93**, 164102 (2004).

[10] H. E. Tureci and A. D. Stone, Opt. Lett. **27**, 7 (2002).

[11] H. M. Lai et al., J. Opt. Soc. Am. A **3** 550 (1986).